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A THEORETICAL BASIS FOR THE LOCKHART–MARTINELLI CORRELATION FOR TWO-PHASE FLOW

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Abstract—Equations are developed in terms of the Lockhart–Martinelli correlating groups for the friction pressure gradient during the flow of gas–liquid or vapour–liquid mixtures in pipes. The theoretical development differs from previous treatments in the method of allowing for the interfacial shear force between the phases; some of the anomalies occurring in previous "lumped flow" models are avoided. Good agreement with Lockhart–Martinelli empirical curves are obtained. Simplified equations for use in engineering design are also given.

NOMENCLATURE

A, cross-sectional area of pipe;
A_G, cross-sectional area of pipe occupied by gas or vapour;
A_i, area of interface between phases per unit length of pipe;
A_L, cross-sectional area of pipe occupied by liquid;
C_G, coefficient in friction coefficient equation for gas or vapour;
C_L, coefficient in friction coefficient equation for liquid;
D, inside diameter of pipe;
D_G, hydraulic diameter of gas flow;
D_L, hydraulic diameter of liquid flow;
f_G, friction factor for gas phase during two-phase flow;
f'_G, friction factor when gas flows alone;
f''_G, friction factor for gas phase in Lockhart–Martinelli theory;
f_i, friction factor at interface between phases;
f_L, friction factor for liquid phase during two-phase flow;
f'_L, friction factor when liquid flows alone;
f''_L, friction factor for liquid phase in Lockhart–Martinelli theory;
K, ratio of gas to liquid velocity;
M_G, mass flowrate of gas;
M_L, mass flowrate of liquid;
m, exponent of Reynolds number in Blasius equation: gas phase;
n, exponent of Reynolds number in Blasius equation; liquid phase;
ΔP_G, friction pressure gradient if gas flows alone;
ΔP_L, friction pressure gradient if liquid flows alone;
ΔP_0, friction pressure gradient if both phases had density of liquid;
ΔP_{TP}, friction pressure gradient during two-phase flow;
Δp, a change in perimeter of a phase at pipe wall;
p_G, perimeter of gas phase at pipe wall;
p_L, perimeter of liquid phase at pipe wall;
S, force between phases per unit length of pipe;
S_R, a "shear force ratio" defined by equation (18);
u_G, velocity of gas during two-phase flow;

u_L, velocity of liquid during two-phase flow;
X, Lockhart–Martinelli parameter, $(\Delta P_L/\Delta P_G)^{\frac{1}{2}}$;
x, ratio of the gas mass flowrate to the total mass flowrate;
Z, dimensionless group defined by equation (29).

Greek symbols
α, ratio of hydraulic diameter of liquid during two-phase flow to that during single-phase flow;
α', as defined by Lockhart and Martinelli: the square of the ratio of the diameter of a pipe of cross-section A_L to the liquid phase hydraulic diameter, equation (3);
β, ratio of hydraulic diameter of gas during two-phase flow to that during single-phase flow;
β', as defined by Lockhart and Martinelli: the square of the ratio of the diameter of a pipe of cross-section A_G to the gas phase hydraulic diameter, equation (4);
θ, angle shown in Fig. 2;
μ_G, absolute viscosity of gas;
μ_L, absolute viscosity of liquid;
ρ_G, density of gas;
ρ_L, density of liquid;
τ_G, shear stress of gas on wall;
τ_i, shear stress between phases;
τ_L, shear stress of liquid on wall;
ϕ_G, Lockhart–Martinelli parameter $(\Delta P_{TP}/\Delta P_G)^{\frac{1}{2}}$;
ϕ_L, Lockhart–Martinelli parameter $(\Delta P_{TP}/\Delta P_L)^{\frac{1}{2}}$.

1. INTRODUCTION

IN RECENT years much of the work on understanding two-phase (gas–liquid or vapour–liquid) flow has been directed towards the examination of particular flow patterns, and considerable progress has been made [1, 2] in this direction. However, the engineer, who may be concerned in the design of plant with the whole range of flow patterns, will normally use more generalized procedures for estimating pressure gradients which are not tied to specific flow patterns. One such procedure is that developed by Lockhart and Martinelli [3] some 18 years ago; over a wide range of conditions this procedure has been shown [4] to give better agreement with experiment than more recent procedures.

In this report the basis of the Lockhart–Martinelli correlation is re-examined.

2. PREVIOUS THEORY

The basis of Lockhart and Martinelli's development [3] is the assumption that for each phase

$$\Delta P_{TP} = 2\, f''_L u_L^2\, \rho_L/D_L \tag{1}$$

and

$$\Delta P_{TP} = 2\, f''_G u_G^2\, \rho_G/D_G, \tag{2}$$

where the phase hydraulic diameters D_L and D_G, the phase velocities U_L and U_G, and the friction factors f''_L and f''_G are defined as follows:

$$A_L = \alpha'\left(\frac{\pi}{4} D_L^2\right), \tag{3}$$

$$A_G = \beta'\left(\frac{\pi}{4} D_G^2\right), \tag{4}$$

$$u_L = \frac{M_L}{\alpha'\left(\frac{\pi}{4} D_L^2\right)\rho_L}, \tag{5}$$

$$u_G = \frac{M_G}{\beta'\left(\frac{\pi}{4} D_G^2\right)\rho_G}, \tag{6}$$

$$f''_L = C_L\left(\frac{4}{\pi}\frac{M_L}{\alpha' D_L \mu_L}\right)^n, \tag{7}$$

and

$$f''_G = C_G\left(\frac{4}{\pi}\frac{M_G}{\beta' D_G \mu_G}\right)^m. \tag{8}$$

As

$$A = A_G + A_L, \tag{9}$$

from equation (3), (4) and (9)

$$\alpha' D_L^2 + \beta' D_G^2 = D^2. \tag{10}$$

The analysis eventually resulted in equations relating

$$\frac{D_L}{D_G}, \quad \frac{D_G}{D}, \quad \alpha' \quad \text{and} \quad \beta'$$

with

$$\phi_L, \quad \phi_G, \quad A_L/A \quad \text{and} \quad A_G/A,$$

where

$$\phi_L = \sqrt{\left(\frac{\Delta P_{TP}}{\Delta P_L}\right)} \tag{11}$$

and

$$\phi_G = \sqrt{\left(\frac{\Delta P_{TP}}{\Delta P_G}\right)}. \tag{12}$$

The analysis also resulted in the postulation that both sets of parameters are functions of X where

$$X = \sqrt{\left(\frac{\Delta P_L}{\Delta P_G}\right)}. \tag{13}$$

Graphical plots of a wide range of data confirmed this postulation and resulted in empirical curves for use in design.

The analysis was unsuccessful in that no equations suitable for predicting pressure gradients were obtained (empirical curves were developed). Also the values of α' obtained were all less than unity, whereas geometric considerations would suggest that α' should be greater than unity; this follows from equation (3) since spread over the wall decreases the hydraulic diameter relative to a circular cross-section.

Turner and Wallis [5] have more recently used essentially the same set of basic equations (1–8) in a treatment which leads to theoretical equations. However, as Wallis writes: "there is unfortunately no rationale for the excellent agreement between (the) equation (relating ϕ_L and A_L/A) and Martinelli's empirical results".

3. PROPOSED THEORY

Irrespective of the flow pattern, the following equations can be written for the force balances on each phase:

$$A_L \Delta P_{TP} - \tau_L p_L + S = 0 \tag{14}$$

and

$$A_G \Delta P_{TP} - \tau_G p_G - S = 0, \tag{15}$$

where the shear force S per unit length of pipe at the interface between the phases acts on the liquid in the direction of motion.

Following normal single-phase procedures the shear forces at the pipe wall will be assumed to be

$$\tau_L = \frac{f_L}{2} u_L^2 \rho_L \tag{16}$$

and

$$\tau_G = \frac{f_G}{2} u_G^2 \rho_G. \tag{17}$$

For convenience define a "shear force ratio"

$$S_R = \frac{S}{A_G \Delta P_{TP}}. \tag{18}$$

Combining equations (14), (16) and (18) gives

$$\Delta P_{TP}\left(1 + S_R \frac{A_G}{A_L}\right) = \frac{f_L u_L^2 p_L \rho_L}{2 A_2} \tag{19}$$

and equations (15), (17) and (18)

$$\Delta P_{TP}(1 - S_R) = \frac{f_G u_G^2 p_G \rho_G}{2 A_G}. \tag{20}$$

In Fig. 1 the assumed phase distribution is indicated and, in Fig. 2, the distribution if the interface between the phases lies along radial lines, the phase cross-sectional areas remaining the same. Let Δp be the change in perimeter of each phase due to this change in distribution,

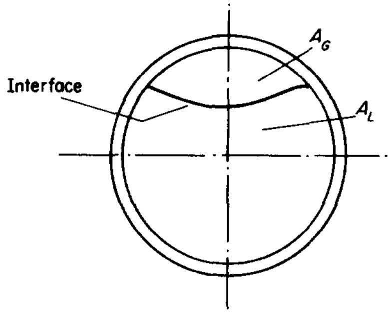

FIG. 1. A representative phase distribution.

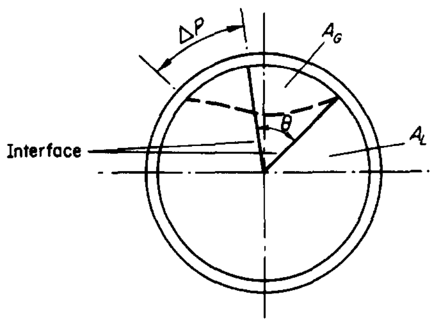

FIG. 2. Phase distribution: each phase occupying a sector of cross-section.

then from geometric considerations

$$\frac{p_G - \Delta p}{A_G} = \frac{4\theta D}{\theta D^2} = \frac{4}{D}, \tag{21}$$

and

$$\frac{p_L + \Delta p}{A_L} = \frac{4}{D}. \tag{22}$$

Now define α and β such that

$$\frac{p_L}{A_L} = \frac{4}{\alpha D}, \tag{23}$$

and

$$\frac{p_G}{A_G} = \frac{4}{\beta D}. \tag{24}$$

Combining equations (9), (21–24),

$$\frac{1}{\beta} = \frac{A}{A_G} - \frac{A_L}{A_G \alpha}. \tag{25}$$

Substituting equations (23) and (24) in equations (19) and (20) respectively gives

$$\Delta P_{TP}\left(1 + S_R \frac{A_G}{A_L}\right) = \frac{2 f_L u_L^2 \rho_L}{\alpha D}, \tag{26}$$

and

$$\Delta P_{TP}(1 - S_R) = \frac{2 f_G u_G^2 \rho_G}{\beta D}. \tag{27}$$

Thus in the proposed theoretical development equations (26) and (27) replace the Lockhart–Martinelli equations (1) and (2).

Combining equations (26) and (27) gives

$$K = u_G/u_L = \frac{1}{Z}\left(\frac{\rho_L}{\rho_G}\right)^{0.5}\left(\frac{\beta}{\alpha}\right)^{0.5}\left(\frac{f_L}{f_G}\right)^{0.5}, \tag{28}$$

where

$$Z = \left(\frac{1 + S_R A_G/A_L}{1 - S_R}\right)^{0.5}. \tag{29}$$

The phase continuity equations are

$$M_L = u_L A_L \rho_L \tag{30}$$

$$M_G = u_G A_G \rho_G, \tag{31}$$

and from equations (28, (30) and (31)

$$\frac{A_L}{A_G} = \frac{1}{Z}\frac{M_L}{M_G}\left(\frac{\rho_G}{\rho_L}\right)^{0.5}\left(\frac{f_L}{f_G}\right)^{0.5}\left(\frac{\beta}{\alpha}\right)^{0.5}. \tag{32}$$

If the phases flow alone, the pressure drops per unit length are given by

$$\Delta p_L = \frac{2 f'_L M_L^2}{D A^2 \rho_L}, \tag{33}$$

and

$$\Delta p_G = \frac{2 f'_G M_G^2}{D A^2 \rho_G}. \tag{34}$$

Combining equations (13, 32–34) gives

$$\frac{A_L}{A_G} = \frac{X}{Z}\left(\frac{f'_G}{f'_2}\right)^{0.5}\left(\frac{f_L}{f_G}\right)^{0.5}\left(\frac{\beta}{\alpha}\right)^{0.5}. \tag{35}$$

and from equations (9, 26, 30, 34)

$$\frac{\Delta P_{TP}}{\Delta P_L} = \frac{1}{\alpha}\frac{f_L}{f'_L}\frac{(1 + A_G/A_L)^2}{1 + S_R A_G/A_L}. \tag{36}$$

Combining this equation with equation (29),

$$\frac{\Delta P_{TP}}{\Delta P_L} = \frac{1}{\alpha}\frac{f_L}{f'_L}\left(1 + \frac{A_G}{A_L}\right)\left(\frac{A_G}{A_L Z^2} + 1\right). \quad (37)$$

4. THE FRICTION FACTORS

The single-phase friction factors f_L and f_G can be expressed

$$f'_L = C_L\left(\frac{A\mu_L}{M_L D}\right)^n, \quad (38)$$

and

$$f'_G = C_G\left(\frac{A\mu_G}{M_G D}\right)^m. \quad (39)$$

It is assumed that the individual phase friction factors during two-phase flow can be expressed

$$f_L = C_L\left(\frac{A_L\mu_L}{\alpha M_L D}\right)^n, \quad (40)$$

and

$$f_G = C_G\left(\frac{A_G\mu_G}{\beta M_G D}\right)^m. \quad (41)$$

The inclusion of α and β in these equations is on the assumption that the characteristic length in Reynolds number is proportional to the hydraulic diameter of the phases.

Hence from equation(38–41)

$$\left(\frac{f'_G\, f_L}{f'_L\, f_G}\right)^{0\cdot 5} = \left(\frac{\beta A}{A_G}\right)^{0\cdot 5m}\left(\frac{A_L}{\alpha A}\right)^{0\cdot 5n}, \quad (42)$$

and from equations (38) and (40)

$$\frac{f_L}{f'_L} = \left(\frac{A_L}{\alpha A}\right)^n. \quad (43)$$

Values of the exponents m and n are given in Table 1. The four flow mechanisms in this table are as defined by Lockhart and Martinelli [3]:

1. Flow of both the liquid and the gas may be turbulent (turbulent–turbulent flow).
2. Flow of the liquid may be viscous and flow of gas may be turbulent (viscous–turbulent flow).
3. Flow of the liquid may be turbulent and flow of the gas viscous (turbulent–viscous flow).
4. Flow of both the liquid and the gas may be viscous (viscous–viscous flow).

It is important to note that equations (42) and (43) are obtained on the assumption that for each phase the single and two-phase flows have the same mechanisms [e.g. the Reynolds number in equations (38) and (40) must in both cases be either turbulent or viscous].

Substituting equations (9) and (42) in (35) gives

$$\left(\frac{A_L}{A}\right)^{1-0\cdot 5n}\left(\frac{A}{A - A_L}\right)^{1-0\cdot 5m} = \frac{X}{Z}\frac{\beta^{0\cdot 5(1+m)}}{\alpha^{0\cdot 5(1+n)}}, \quad (44)$$

and substituting equations (11) and (43) in (37)

$$\phi_L^2 = \frac{1}{\alpha^{1+n}}\left(1 + \frac{A_G}{A_L}\right)^{1-n}\left(\frac{A_G}{A_L Z^2} + 1\right). \quad (45)$$

If Z is known equations (9, 25, 44, 45) can be solved for given values of X and A_L/A to give give ϕ_L. Previous work by the writer [6, 7] on flow through orifices has indicated that for those conditions Z tends to approach a constant value independent of the individual phase flow-rates.

Table 1. Values of m and n

Flow mechanism	turbulent–turbulent		viscous–turbulent	turbulent–viscous	viscous–viscous
Surface	smooth	rough	smooth	smooth	smooth
n	0·2	0	1·0	0·2	1·0
m	0·2	0	0·2	1·0	1·0
Liquid Reynolds number	>2000	>2000	<1000	>2000	<1000
Gas Reynolds number	>2000	>2000	>2000	<2000	<1000

5. COMPARISON WITH EXPERIMENT

Table 2 compares values of ϕ_L predicted using equations (25), (44) and (45) and $Z = 14$ with Lockhart and Martinelli's empirical values. This value of Z was found by trial and error, to give the most satisfactory agreement with the empirical values. In evaluating ϕ_L the values of A/A_L recommended by Lockhart and Martinelli [3] were used; these are shown in Table 2.

Where both phases flow turbulently the predicted values of ϕ_L are within -13 per cent, for a viscous liquid and turbulent gas within -14 per cent, $+21$ per cent; and for both phases flowing viscously -21 per cent, $+16$ per cent. There are no data for the case of a turbulent liquid and a viscous gas.

Table 2. Values of ϕ_L predicted by proposed theory compared with Lockhart–Martinelli values

	ϕ_L			
X	0·1	1·0	10	100
A/A_L	20	4·35	1·88	1·11
Turbulent–turbulent flow				
Lockhart–Martinelli	18·5	4·20	1·75	1·11
Proposed theory $m = n = 0$	18·0	4·02	1·62	1·07
Proposed theory $m = n = 0{\cdot}2$	17·0	3·84	1·53	1·04
Viscous–turbulent flow				
Lockhart–Martinelli	15·2	3·48	1·59	1·11
Proposed theory ($m = 0{\cdot}2$)	18·4	3·63	1·37	1·01
Turbulent–viscous flow				
Lockhart–Martinelli	14·5	3·48	1·66	1·11
Proposed theory ($n = 0{\cdot}2$)	16·2	3·50	1·36	0·952
Viscous–viscous flow				
Lockhart–Martinelli	12·4	2·61	1·50	1·11
Proposed theory	13·2	3·02	1·19	0·88

Figures 3–5 show the comparison of predicted values of ϕ_G with experimental values for the turbulent–turbulent, viscous–turbulent and viscous–viscous regimes; these figures correspond to the figures in Lockhart and Martinelli's original paper (note that $\phi_G = \phi_L X$). The curves of A_L/A obtained by Lockhart and Martinelli

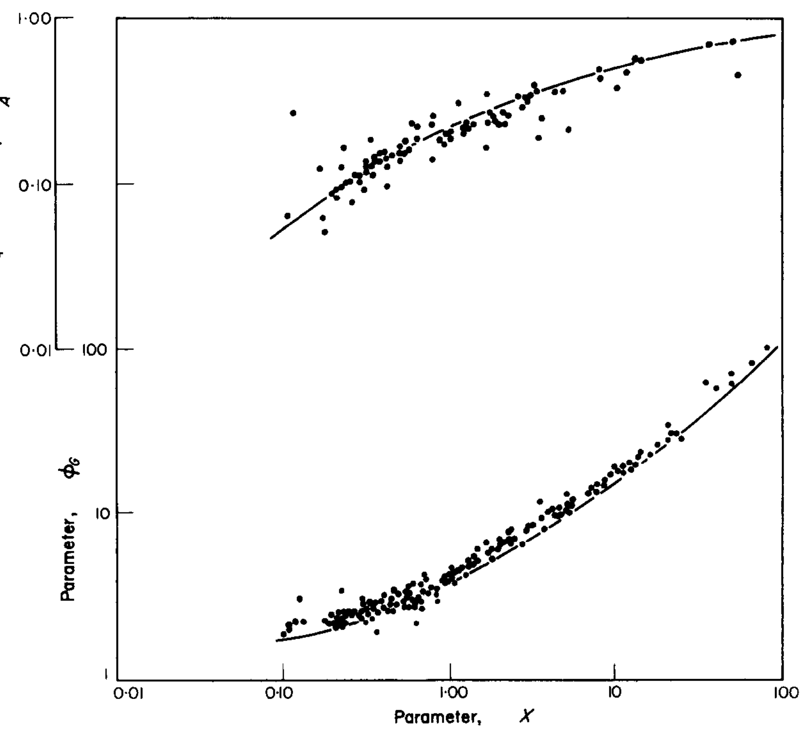

FIG. 3. Relation between A_L/A, ϕ_G and parameter X for turbulent–turbulent flow.

are also shown in Figs. 3–5, the same curve being found to hold for all regimes.

6. ANNULAR FLOW PATTERN AND ZERO SLIP CONDITION

The basis of the above development is the assumption of a value for Z. Two limiting conditions enable Z to be estimated as shown in the Appendix. The first of these is the case of annular flow which results in the equation

$$\phi_L = \left(1 + \frac{A_G}{A_L}\right) \tag{46}$$

for the case of turbulent–turbulent flow in rough tubes ($n = 0$). This form of equation has, of course, been examined elsewhere [8–11].

The zero slip condition, if examined assuming no local slip as in the Bankoff model [12], results in the "homogeneous" equation

$$\frac{\Delta P_{TP}}{\Delta P_0} = \left(1 - x\right) + x\frac{\rho_L}{\rho_G}, \tag{47}$$

or

$$\phi_L = \frac{1}{1 - x}\left\{(1.-.x) + x\frac{\rho_L}{\rho_G}\right\}^{0\cdot5} \tag{48}$$

again for the case of turbulent–turbulent flow in rough tubes ($n = 0$). Table 3 compares values of ϕ_L obtained using equations (45), (46) and (58) with the Lockhart–Martinelli values. Equations (46) and (48) tend to predict values greater than Lockhart and Martinelli, but the annular assumption is not unsatisfactory the maximum difference from Lockhart and Martinelli being +8 per cent; however these equations do not satisfactorily predict values for regimes other than the turbulent–turbulent.

Table 3. Values of ϕ_L by various theories for rough pipes

X	0·1	1·0	10	100
Lockhart–Martinell	18·5	4·2	1·75	1·11
Proposed theory	18·0	4·02	1·62	1·07
Annular theory	20·0	4·35	1·88	1·11
Homogeneous theory	19·3	5·40	1·93	1·10

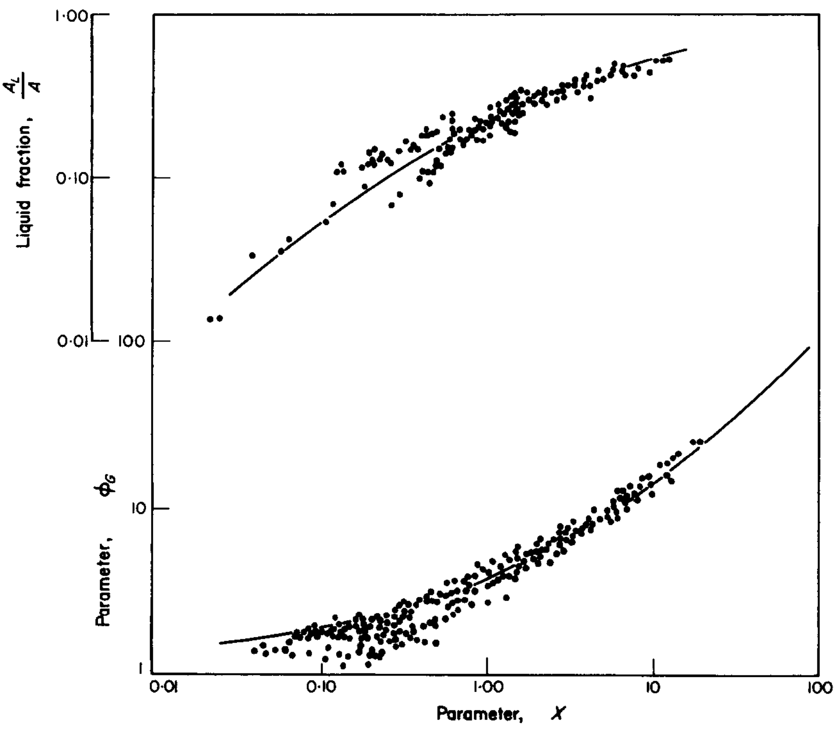

FIG. 4. Relation between A_L/A, ϕ_G and parameter X for viscous–turbulent flow.

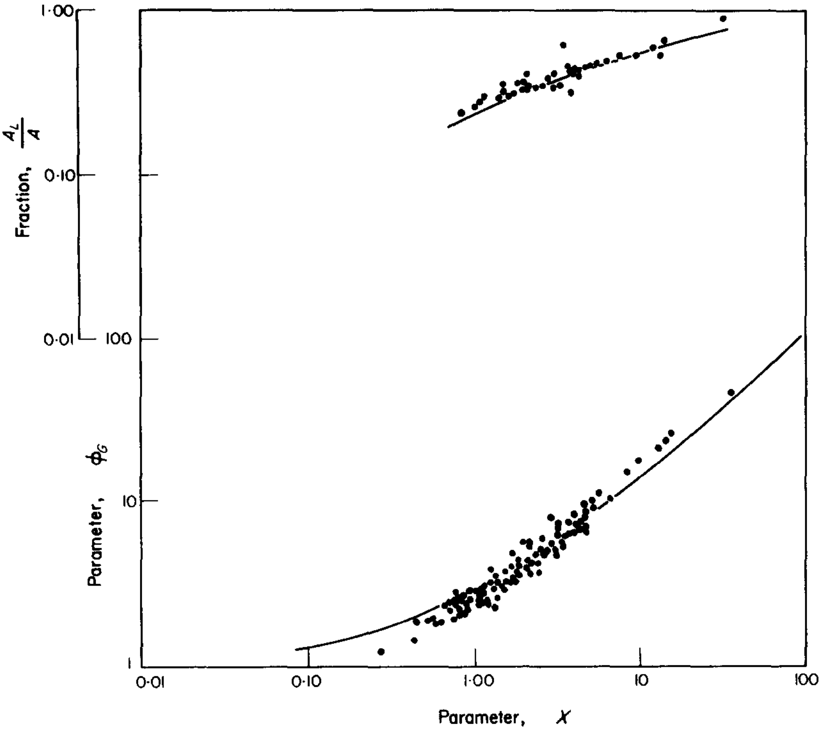

FIG. 5. Relation between A_L/A, ϕ_G and parameter X for viscous–viscous flow.

It is worthwhile examining at this point to which extent the agreement obtained using equation (45) is due to the value of A_L/A selected. In relation to this, consider the data given in [9]. With $X < 2$, A_L/A is a function of liquid mass velocity (i.e. the slip ratio K is a function of M_L [13]) hence for a particular X there are a series of values of A_L/A. Figure 6 compares theory with experiment. In evaluating the predicted curves in this figure, X is kept constant and A/A_L varied; it can be seen that the proposed theory more closely follows the experimental trends than the annular theory.

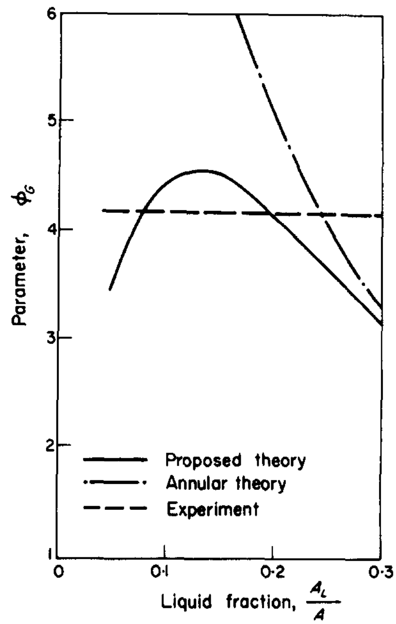

FIG. 6. Variation of ϕ_G with A_L/A for $X = 1$.

7. VALUES OF α AND β

Values of α and β estimated from equations (25) and (44) are given in Table 4. The values of α are generally below unity which is consistent with the known tendency for the heavier liquid phase to approach the wall; the trend of α as

Table 4. Values of α and β

X	0·1		1·0		10		100	
	α	β	α	β	α	β	α	β
Turbulent–turbulent ($m = n = 0$)	0·067	3·67	0·274	4·79	0·719	1·79	0·963	1·53
($m = n = 0{\cdot}2$)	0·071	3·23	0·288	3·82	0·754	1·58	0·998	1·02
Viscous–turbulent	0·057	7·82	0·278	4·48	0·730	1·72	0·995	1·05
Turbulent–viscous	0·077	2·69	0·355	2·46	0·954	1·11	1·172	0·44
Viscous–viscous	0·080	2·56	0·331	2·53	0·846	1·26	1·138	0·48

defined by equation (23) will differ from that of α' defined by equation (5). The present theoretical approach overcomes the anomalous trends in the hydraulic diameter of the liquid phase noted by Lockhart and Martinelli and by Turner and Wallis.

The values of β are normally above unity and it is of interest that the maximum divergences in predicted ϕ_L occurs where β becomes less than unity. This corresponds to the condition where there is a small vapour cross-section distributed, for example, as in Fig. 1, and results in the predicted two-phase pressure drop being less than that for liquid flow alone (the shearing force of the liquid on the wall is greater than in the absence of the gas phase, but the liquid perimeter with the wall has decreased); this is not necessarily physically impossible but is not confirmed by the limited data available in this region.

8. THE SHEAR FORCE FUNCTION Z

Considering now in more detail the function Z defined by equation (29), the interfacial shear force is related to the interfacial shear stress in the equation

$$S = \tau_i A_i. \tag{49}$$

Assume that the interfacial shear stress can be expressed

$$\tau_i = \frac{f_i}{2}(u_G - u_L)^2 \rho_G = \frac{f_i}{2} u_G^2 \left(1 - \frac{1}{K}\right)^2 \rho_G. \tag{50}$$

Substituting equation (50) in (49)

$$S = \frac{f_i}{2} u_G^2 \left(1 - \frac{1}{K}\right)^2 \rho_G A_i. \tag{51}$$

Substituting equations (20), (24) and (51) in (18) gives

$$\frac{S_R}{1 - S_R} = \frac{f_i}{f_G} \frac{\beta D A_i}{4 A_G} \left(1 - \frac{1}{K}\right)^2. \tag{52}$$

Rearranging

$$S_R = \frac{1}{\dfrac{f_G}{f_i'} \dfrac{A_G 4}{\beta D A_i} \dfrac{1}{[1 - (1/K)]^2} + 1}. \tag{53}$$

Substituting equation (53) in (29)

$$Z = \left(\frac{f_i}{f_G} \frac{\beta D}{4} \frac{A A_i}{A_L A_G} \left[1 - \frac{1}{K}\right]^2 + 1\right)^{\frac{1}{2}}. \tag{54}$$

In examining the dimensions of the above expression it should be remembered that A_i is the interfacial surface area per unit length of pipe.

The form of equation (54) gives some explanation for the relative success of treating Z as a constant (it is for example independent of the pressure gradient and to the first order on the phase mass velocities) but more detailed examination of this aspect is required.

9. RECOMMENDED EQUATIONS FOR DESIGN

Equations (44) and (45) are unnecessarily complicated as far as the engineer is concerned. For engineering calculations the writer recommends [9] the following equations for predicting friction pressure drop during two-phase flow in pipes

$$\phi_L^2 = 1 + C/X + 1/x^2, \tag{55}$$

where C as the following values:

turbulent–turbulent flow, $C = 20$;
viscous–turbulent flow, $C = 12$;
turbulent–viscous flow, $C = 10$;
viscous–viscous flow, $C = 5$.

A theoretical basis for equation (55) for turbulent–turbulent flow in rough pipes is given in the Appendix.

Values predicted using these values of C and equation (55) are compared with Lockhart and Martinelli's values in Table 5. The values of C are restricted to mixtures with gas–liquid density ratios corresponding to air–water mixtures at atmospheric pressure. For turbulent–turbulent conditions the writer has discussed elsewhere [14, 15] methods of extrapolating these equations for other density ratios. For the other flow mechanisms further work is required before recommendations can be made on the influence of the density ratio.

Table 5. ϕ_L from equation (55) and Lockhart–Martinelli

	ϕ_L			
X	0·1	1·0	10	100
Turbulent–turbulent flow				
Lockhart–Martinelli	18·5	4·2	1·75	1·11
Equation (55) ($c = 20$)	17·3	4·7	1·73	1·10
Viscous–turbulent flow				
Lockhart–Martinelli	15·2	3·48	1·59	1·11
Equation (55) ($c = 12$)	14·9	3·75	1·49	1·06
Turbulent–viscous flow				
Lockhart–Martinelli	14·5	3·48	1·66	1·11
Equation (55) ($c = 10$)	14·1	3·47	1·42	1·05
Viscous–viscous flow				
Lockhart–Martinelli	12·4	2·61	15·0	1·11
Equation (55) ($c = 5$)	12·3	2·65	12·3	1·03

10. CONCLUSIONS

A theoretical basis for the Lockhart–Martinelli correlating procedure for two-phase flow is developed. This differs from previous developments in the treatment of the interfacial shearing forces between the phases, and results in equations which do not exhibit the anomalous characteristics (e.g. of hydraulic diameter) obtained in previous developments. The equations are also more successful than previous "lumped flow" theories in predicting pressure gradient when one or both phases flow viscously.

A function, Z, of the interfacial shear stresses has been defined. The assumption that this function has a constant value over all the conditions examined by Lockhart and Martinelli gives good agreement between predicted gradients and experiment. The reduction of the equation for limiting values of Z to give equations corresponding to the annular flow and homogeneous flow theories has been demonstrated.

Simplified equations for use in engineering design have been recommended.

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APPENDIX

Annular Flow and Zero Slip Equations

For annular flow

$$p_L = \pi D, \tag{56}$$

hence substituting in equation (23) gives

$$\alpha = \frac{4A_L}{\pi D^2} = A_L/A. \tag{57}$$

From equation (15), as $p_G = 0$

$$S = A_G \Delta P_{TP}. \tag{58}$$

Combining equations (18) and (58) gives

$$S_R = 1; \tag{59}$$

hence from equation (29)

$$Z = \infty. \tag{60}$$

Substitution of equations (9), (57) and (60) in equation (45), where $n = 0$, gives

$$\phi_L = \left(1 + \frac{A_G}{A_L}\right). \tag{61}$$

For the zero slip condition ($K = 1$) if it is assumed that there is also no local slip, then, using the reasoning forming the basis of the variable density model of Bankoff [11], the phase densities must also be uniformly distributed radically. In that case the phase perimeter is proportional to the phase cross-sectional area. From equations (23) and (24) therefore, α and β will both be unity. As rough tubes are being considered f_L and f_G will be identical. Hence from equation (28)

$$Z = \left(\frac{\rho_L}{\rho_G}\right)^{0.5}, \tag{62}$$

and from equation (32)

$$\frac{A_L}{A_G} = \frac{1}{Z}\frac{M_L}{M_G}\left(\frac{\rho_G}{\rho_L}\right)^{0.5} = \frac{1}{Z}\frac{1-x}{x}\left(\frac{\rho_G}{\rho_L}\right)^{0.5}. \tag{63}$$

For rough tubes it is readily shown that

$$X = \sqrt{(\Delta P_L/\Delta P_G)} = \frac{1-x}{x}\left(\frac{\rho_G}{\rho_L}\right)^{0.5}. \tag{64}$$

Substituting equation (64) in (63),

$$\frac{A_L}{A_G} = \frac{X}{Z}. \tag{65}$$

Combining equation (45) and (65), where $\alpha = 1$ and $n = 0$, gives

$$\phi_L^2 = 1 + C/X + 1/X^2, \tag{66}$$

where

$$C = Z + \frac{1}{Z}. \tag{67}$$

Combining equations (62), (64), (66) and (67) gives

$$\phi_L = \frac{1}{(1-x)}\left\{(1-x) + x\frac{\rho_L}{\rho_G}\right\}^{0.5}; \tag{68}$$

also as

$$\Delta P_0 = \Delta P_L/(1-x)^2, \tag{69}$$

it follows that

$$\frac{\Delta P_{TP}}{\Delta P_0} = (1-x) + x\frac{\rho_L}{\rho_G}. \tag{70}$$

Résumé—On expose des équations utilisant les groupes de corrélation de Lockhart–Martinelli pour exprimer le gradient de pression dû au frottement pendant l'écoulement dans des tuyaux de mélanges gaz–liquide ou vapeur–liquide. Le développement théorique diffère des méthodes antérieures en ce que l'on tient compte des forces de cisaillement aux interfaces entre les phases; quelques-unes des anomalies qui se produisent dans des modèles antérieurs d'"écoulement global" sont évitées. On obtient un bon accord avec les courbes empiriques de Lockhart–Martinelli. On donne également des équations simplifiées destinées à être employées dans la technique.

Zusammenfassung—Es wurden Gleichungen entwickelt in der Art der Lockhart–Martinelli-Beziehungen, welche die Ausdrücke für den Reibungsdruckgradienten der Strömung von Gas–Flüssigkeits- oder Dampf–Flüssigkeitsgemischen in Rohren korrelieren. Die theoretische Behandlung unterscheidet sich von früheren Entwicklungen in der Methode, nach der die Reibungskräfte in der Grenzschicht zwischen den Phasen berücksichtigt wurden; einige der Anomalien der früheren Modelle der "Klumpenströmung" verschwinden hier. Gute Übereinstimmung ergibt sich mit den empirischen Kurven von Lockhart–Martinelli. Vereinfachte Gleichungen für ingenieurmässige Anwendungen sind ebenfalls angegeben.

Аннотация—На основе корреляционных групп Локхарта-Мартинелли предложены уравнения для градиента давления и трения при течении газожидкостных и парожидкостных смесей в трубах. Теоретический подход отличен от предыдущего метода, базирующегося на допущении о существовании сдвиговых напряжений на поверхности раздела фаз. Исключены некоторые аномалии, встречающиеся в предыдущих моделях «массивного потока». Получено хорошее согласование с эмпирическими кривыми Локхарта–Марттнелли. Даются также упрощенные уррвнеъия для применения в технических расчетах.